

Math 207: Elementary Linear Algebra with Applications
Homework 2

1. Consider the following system of equations

$$\begin{aligned}2a + 2b - 2c - 3d + 9e - 7g &= 1 \\2a + 2b - 4c - 6d + 8e - 6g &= 0 \\a + b + d + 4e - 4g &= 1\end{aligned}$$

- (a) Write down the augmented matrix of the system.
- (b) Write down the system in matrix form identifying the coefficient matrix and the matrix of unknowns.
- (c) Reduce the augmented matrix to echelon form identifying clearly all the elementary operations.
- (d) Find the basic solutions.
- (e) Find all solutions. Express them in vector form.

2. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 4 \\ -2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Compute each of the following, if possible. If not possible, explain why not.

(a) AB (b) BA (c) $A + B$ (d) AC (e) CB (f) $C^t B$ (g) CC^t (h) $C^t C$.

3. Suppose $A = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$ and $AB = \begin{pmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{pmatrix}$. What is B ?

4. Find all 2×2 matrices whose square is the identity. (Careful! There are many such matrices.)

5. How many algebraic operations does it take to multiply an $n \times m$ matrix by an $m \times r$ matrix? Explain your answer fully.

6. True/False: If A, B, C are 2×2 matrices with the property that $AB = AC$, then $B = C$. If true, explain why; if false, give a counterexample.

7. Find all 2×2 matrices A with the property that $A \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. (Of, course, there may not be any that work.)