Math 207: Elementary Linear Algebra with Applications
Homework 4

1. (a) Compute the inverse of $A = \begin{pmatrix} 1 & -2 & -4 \\ -3 & -2 & 3 \\ 2 & -1 & -5 \end{pmatrix}$.

(b) Use this inverse to solve $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 19 \\ 10 \\ 14 \end{pmatrix}$.

2. When is an upper triangular matrix invertible? Explain your answer.

3. Let A_n be the $n \times n$ matrix with 0's on the main diagonal and 1's everywhere else; for example

$$A_3 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

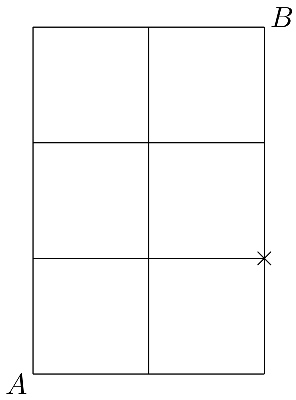
Compute the inverse of A_n for $n = 2, 3, 4$. Make a conjecture as to the general form of the inverse of A_n for arbitrary values of n.
(Bonus: Prove your conjecture.)

4. Suppose A, B, C are non-zero 2×2 matrices and $AB = AC$. Determine whether each of the following statements is true or false. If true, explain why; if false, give a counterexample.
(a) If A is invertible, then $B = C$.
(b) If B is invertible, then $B = C$.
(c) If C is invertible, then $B = C$.

5. (a) Express the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ as a product of elementary matrices.
(b) Explain why every invertible matrix is a product of elementary matrices. Describe an algorithm which expresses an arbitrary invertible matrix as such a product.

6. The Demographic Research Unit of the California State Department of Finance has determined that in 1989 about 11.7% of the U.S. population lived in California. They also found that 98.21% of the California population stayed in the state, and the remaining 1.79% of the California population moved to elsewhere in the U.S.; 0.29% of the U.S. population outside California migrated into the state, while the remaining 99.71% of the U.S. population remained outside California. If the migration probabilities were to remain constant over many years, what percent of the U.S. population would eventually live in California?

7. An ant is placed on a 2×3 grid at the position marked $\times$ on the right. The ant moves along the edges of the grid using the following rule: From an intersection it randomly chooses one of the edges radiating out and moves along it arriving at the next intersection one "step" later. (Note that the ant is allowed to double back on its path.) The points marked A and B are exits; when the ant reaches one of these, it leaves the grid forever. What is the probability that it will exit at point A? Is the ant more likely to exit through point A or point B? How much more likely?



8. Suppose an experiment leads to the following SOE:

$$\begin{aligned} 4.5\,x + 3.1\,y &= 19.249 \\ 1.6\,x + 1.1\,y &= 6.843 \end{aligned}$$

(a) Solve the system for x and y exactly.

Next consider the system above with the numbers rounded to two decimal places.

$$\begin{aligned} 4.5\,x + 3.1\,y &= 19.25 \\ 1.6\,x + 1.1\,y &= 6.84 \end{aligned}$$

(b) Solve this system for x and y exactly.
(c) How reliable is the solution of the rounded system as an approximation of the original system?