Math 207: Elementary Linear Algebra with Applications
Homework 5

1. (a) Find the line of best fit to the data $(0,0),\ (1,1),\ (2,3)$. Label the matrix of coefficients for the normal equations.
(b) Find a polynomial of degree 4 of best fit to the data points

$$(1,1),\ (2,1^3+2^3),\ (3,1^3+2^3+3^3),\ (4,1^3+2^3+3^3+4^3),\ (5,1^3+2^3+3^3+4^3+5^3).$$

Explain your work. (You may use your calculator to solve the relevant SOE, but you must exhibit the SOE and explain where it came from.)

2. The table below shows the mean distance, D, and the period, P, of the (known) planets in our solar system. Assuming a relationship of the form $D^a = P^b$, find a and b by using a least squares procedure.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distance from sun ($\times 10^6$ km)	57.9	108	150	228	778	1,430	2,870	4,500	5,900
Period of revolution (earth years)	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248

(Hint: Given the supposed model, what should the data look like on a log-log graph? Modify the data accordingly and find the corresponding parameters of the modified equation.)

3. Suppose the SOE $Ax = b$ has a unique solution for each vector b. Explain why A is invertible.

4. Suppose A is an $n \times n$ matrix. Explain why the following two statements are true.
(a) Whenever A is not invertible, then the homogeneous SOE $Ax = 0$ has infinitely many solutions.
(b) Whenever the homogeneous SOE $Ax = 0$ has infinitely many solutions, then A must be non-invertible.

5. Suppose that the oldest age attained by the females in an animal population is 15 years. Divide the population into three equal age groups of 5 years each. Suppose the Leslie matrix for this population is

$$L = \begin{pmatrix} 0 & 4 & 3 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \end{pmatrix}$$

(a) What information does the (1,2)–entry of L convey? What about the (3,2)–entry?
(b) If there are initially 1000 females in each of the three age groups, how many females are there in each age group after 5 years? After 10 years? After 15 years?

6. Let M be the transition matrix for a Markov chain with n states. Explain why $M^t v = 0$, where v is the $n \times 1$ vector of all ones.

7. The Land of Oz is meteorologically cursed; indeed, its inhabitants never experience two nice days in a row. A nice day is just as likely to be followed by snow as by rain the next day. Whenever they have snow or rain, they have an even chance of having the same the next day. A nice day follows a snowy day only half the time, with the same being true after a rainy day.
(a) Model this data with a Markov chain. Write down the (carefully labelled!) transition matrix.
(b) Is this Markov chain regular? How can you tell?
(c) What is the long-term weather forecast in the Land of Oz? Give the precise probability of a nice day, a rainy day, a snowy day.