1. Given points \( P(1,2,0), Q(3,4,3), R(-5,0,1) \) find an equation of a plane which contains these three points.

Solution: for a point use any of these; get a normal

\[
\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & 2 & 3 \\ -6 & -2 & 1 \end{vmatrix} = \langle 2+6-(-2+18), (-4+12) \rangle = \langle 8, -20, 8 \rangle = \overrightarrow{n}
\]

So \( 8(x-1) - 20(y-2) + 8z = 0 \) works; so does \( 2(x-1) - 5(y-2) + 2z = 0 \).

Check: plugging in \( P \) is obvious; plug in \( Q : 2(3-1) - 5(4-2) + 2 + 3 = 4 - 10 + 6 = 0 \)

plug in \( R : 2(-5-1) - 5(-2) + 2 = -12 + 10 + 2 = 0 \).

2. If possible, find where the two following lines intersect, and find an equation of the plane which contains both lines. If the lines don’t intersect, say why.

Line 1: \(-6,1,5 > +t < 4,0,-1 \)

Line 2: \(< 3,0,7 > +s < 1,-1,4 > \)

Equate the coordinates to see if the lines intersect:

\[
\begin{align*}
x &= -6 + 4t = 3 + s, \\
y &= 1 = -s, \\
z &= 5 - t = 7 + 4s;
\end{align*}
\]

It isn’t hard to see that \( s = -1 \) and \( -6 + 4t = 3 - 1 \) and \( -6 + 4t = 2 \) so \( 4t = 8 \) or \( t = 2 \).

Does this work for \( z \)? \( 5 - 2 = 7 - 4? \) Yes. So, the lines intersect (at point \( (2,1,3) \))

Now we need a normal vector; use \( < 4,0,-1 > \times < 1,-1,4 > = \langle \text{det} \rangle \)

\[
\begin{vmatrix} i & j & k \\ 4 & 0 & -1 \\ 1 & -1 & 4 \end{vmatrix} = \langle 1, -1, 4 \rangle = < -1, -15, -4 >
\]

\( < -1, -15, -4 > \) so an equation is \( -(x - 3) - 17y - 4(z - 7) = 0 \) (here I used \( 3,0,7 \) as a point; other points would work. Another equation might be \( x - 3 + 17y + 4(z - 7) = 0 \).

Check: plug in \( (2,1,3) : 2 - 3 + 17 + 4(3 - 7) = 0 \); this works. Plug in \( (-6,1,5) : -9 + 17 - 8 = 0 \); that works; plug in \( (3,0,7) : 0 + 0 + 0 = 0 \); that works. Three points determine a plane so we know we are right.

3. Find \( \overrightarrow{PQ} \) if \( \overrightarrow{a} = 2\overrightarrow{i} - \overrightarrow{j} + 3\overrightarrow{k} \) and \( \overrightarrow{b} = -\overrightarrow{i} + \overrightarrow{j} - \overrightarrow{k} \)

\[
\left( \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{||\overrightarrow{a}||} \right) = \left( \frac{2i - 1j + 3k}{4} \right) = \frac{1}{\sqrt{14}} \left( \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{||\overrightarrow{a}||} \right) = \frac{6}{\sqrt{14}} < 2, 1, 3 > = \frac{-6}{7} < 2, -1, 3 > = \frac{-6}{7} < \frac{3}{7}, -\frac{9}{7} >
\]

4. Find the angle between \( \overrightarrow{a} = < 4,0,-1 > \) and \( \overrightarrow{b} = < 1,-1,4 > \).

\[
\overrightarrow{a} \cdot \overrightarrow{b} = 4 - 4 = 0 \rightarrow \theta = \frac{\pi}{2}
\]
5-7. Suppose we have the following equation: \( y = 4x^2 + z^2 - 8z + 16 \)

5. Make a quick sketch of the traces \( y = 0, y = 1, y = 4, y = -1 \). Sketch these traces in the \( x,z \) plane.
First write this as \( y = 4x^2 + (z - 4)^2 \) the \( y = 0 \) trace is merely the point \((0,4)\); the \( y = 1 \) trace is the ellipse \( 1 = 4x^2 + (z - 4)^2 \); the \( y = 4 \) trace is the ellipse \( 4 = 4x^2 + (z - 4)^2 \) and the \( y = -1 \) trace is empty (\( y \) is never negative).

6. Make a quick sketch of the \( x = 0 \) trace in the \( y,z \) plane.
\( y = (z - 4)^2 \) which is an upwards opening parabola, with vertex at \( y = 0, z = 4 \).

7. What kind of surface this? Do your best to make a sketch in 3-space \((x,y,z)\) space.
This is an elliptic paraboloid which runs parallel to the \( y \) axis.

8. If \( f(x,y) = \frac{\sqrt{x^2 + 3y^2 - 1}}{\ln(xy)} \) find the domain for \( f \).
Domain: first we need \( xy > 0 \rightarrow \) either \( x > 0 \) and \( y > 0 \) or \( x < 0 \) and \( y < 0 \); hence we are in quadrants 1 and 3. Next we need \( xy \neq 1 \) (\( \ln(1) = 0 \)) so we must throw out the graph of \( y = \frac{1}{x} \).
Next we need \( x^2 + 3y^2 - 1 \geq 0 \rightarrow x^2 + 3y^2 \geq 1 \) → we need to be outside of the ellipse \( x^2 + 3y^2 = 1 \) as well.

So, formally, the domain is \( \{(x,y)|x > 0 \text{ and } y > 0 \text{ or } x < 0 \text{ and } y < 0, y \neq \frac{1}{x}, x^2 + 3y^2 \neq 1\} \)

9. Find the volume of the parallelepiped determined by the following vectors:
\( \vec{i} + \vec{j}, 2\vec{i} - \vec{k}, 3\vec{j} + \vec{k} \).
Solution: \( V = |\text{det} \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 3 & 1 \end{bmatrix}| = 3 - 2 = 1. \)

10. Suppose we are given the equations: \( x(t) = 2\cos(t) - 1, y(t) = 3\sin(t), 0 \leq t < 2\pi \).
What curve does this define? In which direction is the curve parametrized?
This is parametrized in the standard direction; the curve:
\( x + 1 = 2\cos(t) \rightarrow \frac{x+1}{2} = \cos(t), \frac{y}{3} = \sin(t) \rightarrow (\frac{x+1}{2} )^2 + (\frac{y}{3} )^2 = 1 \)
This is an ellipse centered at \((-1, 0)\).

11. Find an equation of the plane which is perpendicular to the line which has the following equation:
\[
\frac{x-3}{2} = \frac{y+1}{5} = z - 3
\]
and contains the point \(P(2, 1, 3)\).

We are given a point for free; we need a normal vector (the direction vector of the line will do)
Let \(t = \frac{x-3}{2} \rightarrow x = 3 + 2t, \frac{y+1}{5} = t \rightarrow -1 + 5t = y, z = t + 3\)
So the line has direction vector \(<2, 5, 1>\) and so an equation of the plane is:
\[2(x - 2) + 5(y - 1) + (z - 3) = 0.\]

12. Write the parametric equations of a line segment which goes from \(P(-1, 2)\) to \(Q(0, 5)\) as the parameter \(t\) ranges from 0 to 1.
\[x = -1 + (0 - (-1))t, y = 2 + (5 - 2)t \rightarrow x = -1 + t, y = 2 + 3t.\]