7.2 \( \sigma = 2, P(|\bar{Y} - \mu| < .3) = P\left(\frac{|\bar{Y} - \mu|}{\sigma / \sqrt{n}} < \frac{.3}{\sigma / \sqrt{n}}\right) = P(|Z| < .45) = P(|Z| < .45) = 1 - 2 \cdot .3264 = .3472 \)

Note that when we use \( n = 25 \), \( P(|Z| < \frac{.3}{\sqrt{25}}) = P(|Z| < .9) = 1 - 2 \cdot .1841 = .6318 \),

\( n = 36 \), \( P(|Z| < \frac{.3}{\sqrt{36}}) = P(|Z| < .5) = 1 - 2 \cdot .1587 = .6813 \),

\( n = 49 \), \( P(|Z| < \frac{.3}{\sqrt{49}}) = P(|Z| < .15) = 1 - 2 \cdot .1469 = .7062 \),

\( n = 64 \), \( P(|Z| < \frac{.3}{\sqrt{64}}) = P(|Z| < 1.2) = 1 - 2 \cdot .1151 = .7698 \)

The probability goes up as the sample size goes up. Though the following calculations are not required to complete the problem, note what happens when \( \sigma = 1 \) (is cut in half)

\( n = 25 \), \( P(|Z| < \frac{.3}{\sqrt{25}}) = P(|Z| < 1.5) = 1 - 2 \cdot .0668 = .8664 \)

\( n = 36 \), \( P(|Z| < \frac{.3}{\sqrt{36}}) = P(|Z| < 1.8) = 1 - 2 \cdot .0359 = .9282 \),

\( n = 49 \), \( P(|Z| < \frac{.3}{\sqrt{49}}) = P(|Z| < 2.1) = 1 - 2 \cdot .0179 = .9642 \),

\( n = 64 \), \( P(|Z| < \frac{.3}{\sqrt{64}}) = P(|Z| < 2.4) = 1 - 2 \cdot .0082 = .9836 \)

Less variation in the population means more certainty in the results.

7.4 \( P(|\bar{Y} - \mu| < 1) = .9 \rightarrow P\left(\frac{|\bar{Y} - \mu|}{\sigma / \sqrt{n}} < \frac{1}{\sigma / \sqrt{n}} \times 1\right) = .9 \) but we know that \( P(|Z| < 1.645) = .9 \)

So \( \frac{\sigma}{\sqrt{n}} \times 1 = 1.645 \rightarrow \sqrt{n} = .645 \times \sigma = 6.58 \rightarrow n \geq 43.3 \rightarrow n = 44 \)

7.6 Variance = .4 \( \rightarrow \sigma = .6325 \); want \( n \) so that \( P(|\bar{Y} - \mu| < .5) \); note

\( P(|Z| < Z_{.025}) = .95 \rightarrow Z_{.025} = 1.96 \)

\( P\left(\frac{|\bar{Y} - \mu|}{.6325} < 1.96\right) = .95 \) and we want

1.96 \( \cdot .6325 / \sqrt{n} = .5 \rightarrow \sqrt{n} = 1.96 \cdot .6325 / .5 = 2.481 \rightarrow n = 6.16 \)

Use \( n = 7 \).

7.12 Want \( P(g_1 \leq (\bar{Y} - \mu) \leq g_2) = .9 \); note that we have a standard deviation estimated from the data; \( S = 4 \) \( \rightarrow \) use the "t" distribution with \( n - 1 = 8 \) degrees of freedom;

\( P(-1.860 < T < 1.860) = .9 \)

\( T = \frac{\bar{X} - \mu}{s / \sqrt{n}} \)

\( P(-1.860 < \frac{\bar{X} - \mu}{s / \sqrt{n}} < 1.860) = .9 \rightarrow P\left(-\frac{.4}{s / \sqrt{n}} \cdot 1.860 < \frac{\bar{X} - \mu}{s / \sqrt{n}} < \frac{.4}{s / \sqrt{n}} \cdot 1.860\right) = .9 \rightarrow g_i = \pm 2.48 \) as appropriate.

7.20 a) Chi-squared, 5 degrees of freedom.

b) Chi-squared, 4 degrees of freedom.

c) Note that \( \bar{Y} \) is normal with variance \( \frac{1}{\sqrt{5}} \rightarrow \sqrt{5} \bar{Y} \) is normal with variance 1.

So \((\sqrt{5} \bar{Y})^2 = 5 \bar{Y}^2 \) is Chi-squared with 1 degree of freedom. \( \bar{Y}^2 \) is Chi-squared with 1 degree of freedom. Since \( \bar{Y}^2 \) is independent from \( \bar{Y} \), the sum of these two Chi-squared random variables is chi-squared with 2 degrees of freedom. \( U \) is Chi-squared with 4 degrees of freedom, and independent from both of the previous random variables. So look at Definition

7.3: \( F_{v_1}^{v_2} = \frac{w_1}{v_1} \frac{v_1}{w_1} \rightarrow v_1 = 2 \) and \( v_2 = 4 \rightarrow \frac{v_2}{w} = 2 \rightarrow \)
\[ \frac{2(5\bar{Y} + 12)}{U} \] is \( F \) with 2 numerator and 4 denominator degrees of freedom.

7.22

\[ P(\bar{Y} > 14.5) = P(\bar{Y} - 14 > 14.5 - 14) = P(\bar{Y} - 14 > .5) = P(\frac{\bar{Y} - 14}{\frac{5}{\sqrt{400}}} > \frac{5}{\frac{1}{\sqrt{400}}} ) = P(Z > \frac{5 \times 10}{2}) = \]

\[ P(Z > 2.5) = .0062 \] (unlikely; with a sample this large that measured this much we would conclude that the sample mean is NOT 14.0)

b) \( P(|Z| < 1.96) = .95 \)

\[ 1.96 \times \frac{2}{\sqrt{100}} \geq |\bar{Y} - \mu| \rightarrow P(|\bar{Y} - \mu| < .392) = .95 \rightarrow P(13.608 < \bar{Y} < 14.392) = .95 \]

7.28 \[ P(|\bar{Y} - \mu| < 1) = P(\frac{|\bar{Y} - \mu|}{\frac{12}{\sqrt{35}}} < \sqrt{\frac{15}{12}} ) = P(|Z| < .493) = 1 - \frac{1}{2} \times .3121 = .3758 \]

b) No; there is no reason to believe that it is even likely.

7.30. \( P(|\bar{Y} - \mu| < 1) = P\left( \frac{|\bar{Y} - \mu|}{\frac{12}{\sqrt{35}}} < \sqrt{\frac{15}{12}} \right) = P(|Z| < 1) = 1 - \frac{1}{2} \times .1587 = .6826 \) (remind you of the empirical rule?)

7.36. We need \( P(50 \times \bar{Y} > 200) \geq .95 \rightarrow P(\bar{Y} > 4) \geq .95 \rightarrow P(\bar{Y} - \mu > 4 - \mu) \geq .95 \)

\[ P(\frac{\bar{Y} - \mu}{\frac{4 \times \mu}{\sqrt{50}}} \geq .95 \rightarrow P(Z > \frac{4 - \mu}{\sqrt{50}}) \geq .95 \) (note: variance is 4 so \( \sigma = 2 \))

\[ \frac{4 - \mu}{\sqrt{50}} = -1.645 \rightarrow \mu = 4.4653 \]
7.24, 7.26, 7.32, 7.34, 7.42

7.24
\[ P(|Z| < 1.96) = .05 \rightarrow P(\left| \frac{\bar{Y} - \mu}{\sigma / \sqrt{n}} \right| < 1.96) \rightarrow P(|.4| < 1.96 \times \frac{.25}{\sqrt{n}}) \rightarrow \]
\[ \sqrt{n} = \frac{1.96 \times .25}{.4} = n = 151 \]

7.26 \( \sigma \approx \frac{8.5}{4} = 0.75; P(|\bar{Y} - \mu| < .2) = P\left(\left| \frac{\bar{Y} - \mu}{\sigma / \sqrt{n}} \right| < \frac{2}{\sqrt{75}} \right) \approx P(|Z| < 1.69) = 1 - 2 \times (.0455) = .909 \]

7.32 a) \[ P\left( \frac{199 - 200}{10} < Z < \frac{202 - 200}{10} \right) = P\left( \frac{-5}{10} < Z < \frac{2}{10} \right) = P(-.5 < Z < 1) = 1 - .3085 - .1587 = .5328 \]

b) \[ P(25 \bar{Y} < 5100) = P(\bar{Y} < \frac{5100}{25}) = P\left( \bar{Y} - \frac{200}{25} < \frac{3100 - 200}{25} \right) = P(Z < \frac{4}{2}) = 1 - .0228 = .9772 \]

7.34 \[ P(\bar{Y} < 1.3) = P\left( \frac{\bar{Y} - 1.4}{.05} < \frac{1.3 - 1.4}{.05} \right) = P(Z < -10) \approx 0. \]

7.42 \[ P(\sum_{i=1}^{100} Y_i \geq 4 \times 60) = P\left( \frac{1}{100} \sum_{i=1}^{100} Y_i \geq \frac{1}{100} \times 240 \right) = P(\bar{Y} \geq 2.4) = P\left( \frac{\bar{Y} - 2.5}{\sqrt{100}} > \frac{2.4 - 2.5}{\sqrt{100}} \right) = P(Z > \frac{-1}{2}) = P(Z > -0.5) = 1 - .3085 = .6915 \]