Homework three: 2.68, 2.74, 2.78, 2.86, 2.90, 2.94

2.68
a) \((.05)^3 = .000125\)
b) \(1 - (.95)^3 = .142625\)

2.74 Series: \(P(O) = (.9)^2 = .81\)
Parallel: \(P(O) = 1 - P(\overline{O}) = 1 - (.1)^2 = .99\)

2.78 a) \(P(I \cup II) = P(I) + P(II) - P(I \cap II) = P(I) + P(II) - P(I)P(II) = .1 + .15 - .03 = .22\)
b) \(P(I \cap II | I \cup II) = \frac{P(I \cap II)}{P(I \cup II)} = \frac{.03}{.22} = .13636\)

2.86 \(P(N) = P(N|I)P(I) + P(N|II)P(II) = .08 \times .4 + .1 \times .6 = .092; P(D) = 1 - .092 = .918\)

2.90 Let \(L\) denote the liar and \(T\) denote the truth teller; let \(+\) denote a positive test (that is, the test registers a lie).
a) \(P((+|T) \cap (+|L)) = P(+|T) \times P(+|L) = .1 \times .95 = .095\)
b) \(P((+|L) \cap (-|T)) = P(+|L) \times P(-|T) = .95 \times .9 = .855\)
c) \(P((-|L) \cap (+|T)) = P(-|L) \times P(+|T) = .05 \times .1 = .005\)
d) \(1 - P((-|L) \cup (-|T)) = 1 - .05 \times .9 = .955\) Note: since a, b, c are mutually exclusive events and partition all the cases in which there is at least one positive reading, you can add a, b, and c together to get d.

2.94 The victim can afford to have at most 4 tests. The probability that there is no match in the first 4 tests is as follows: \((.6)^4 = .1296\); So, the probability that the victim will live is \(1 - .1296 = .8704\)